Multiple Carrier-Vehicle Travelling Salesman Problem

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Abstract. In this paper the Carrier-Vehicle Travelling Salesman Problem (CV-TSP) is extended to the case of 2 carriers and one small vehicle. The paper defines a minimum-time trajectory mission plan for the visit of a group of target points by the small vehicle. In this scenario the main goal is to optimize the use of both carriers as a support of the vehicle. A Mixed-Integer Second Order Conic Programming (MISCOP) formulation is proposed for the case of a given order of visit. Additionally, the authors develop a fast heuristic which provides close to optimal results in a decent computational time. To end the paper several simulations are computed to show the effectiveness of the proposed solution.

1 Introduction

Over the past few years, the use of autonomous systems is experiencing a tremendous rise. As a result, the tasks and applications envisioned for this kind of systems are gaining in complexity and importance. Current rescue missions, logistics and transportation activities require such a wide range of capabilities -large autonomy, small size and maneuverability- that they cannot be provided by a single class of vehicle. Alternatively, the combination of different class platforms represents a more adequate solution to reach the specifications demanded [1].

While the coordination of several units of homogeneous vehicles has been widely developed and many complex applications are already established [2, 3], the research work in heterogeneous systems is still at an early stage. In recent years, different research groups have studied the Traveling Salesman Problem applied to a team of heterogeneous vehicles. The Multi-Depot Heterogeneous Fleet Routing Vehicle Problem [4] considers the use of vehicles with different capacities and speeds to solve a routing problem. In [5], influenced by the current rise of drone applications, the last mile delivery problem is solved using a TSP approach for a group of drones while deployed by a truck with a fixed route.

Another recent contribution to this field is the Carrier-Vehicle Travelling Salesman Problem (CV-TSP) [6]. This variant of the TSP presents as a novelty

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the use of two different vehicles, a fast small vehicle and a slow carrier, which are combined to perform different missions. The CV-TSP considers the scenario where a fast vehicle with limited endurance is transported and serviced by a slower carrier. The authors define a mathematical model of a carrier-vehicle system dynamics and the associated constraints. An optimal trajectory calculation connecting two points is provided, followed by a generalization of the problem to the visit of set of points obtaining a first sub-optimal solution. In [7] a first exact solution method for the CV-TSP was proposed. This problem, originally thought for rescue missions, rapidly demonstrated the applicability in other fields as logistics or transportation problems[8, 9].

More recently, [10] extends the CV-TSP and proposes the case of 2 vehicles and one carrier. This extension of the original problem is motivated by cooperative search and reconnaissance missions in heterogeneous robot systems. Moreover, it can be easily proved that a larger group of heterogeneous vehicles for visiting a given set of target points will always result in a solution (i.e. total travel time) lower or equal than the one of a heterogeneous team of 2 vehicles. This work also provides a first non-linear formulation and good results using a deep learning approach.

In this paper we introduce a novel extension of the CV-TSP to the case of two carriers and one single vehicle. In the described scenario, the small vehicle can choose indistinctly between both carriers to land and be serviced. Equivalently to [10], this variant always provides a faster mission time that the original case. It should be noticed that this complementary extension remains interesting as it includes a whole new group of additional applications. Such as the case of maritime monitoring, where we can find examples of single UAVs with multiple cruise bases [11] or mobile self powered carriers with a single UAV flying from one to another [12]. Another application example is encountered in the EU project PANTHEON "Precision farming of hazelnut orchards". In this case, the UAVs used for the orchard coverage have a limited autonomy and data storage. There, the coordination in movement between an aerial vehicle and larger ground robots to charge batteries and download data would increase the scalability of the concept to large-scale plantations [13].

The remainder of the paper is organized as follows. In Section 2 the problem is defined and an optimal mixed-integer formulation is proposed. In Section 3 a fast heuristic is presented for the case of large inputs. In Section 4 several numerical results are shown. In Section 5 we finish with some conclusions and future work.

2 Problem statement

The system studied is composed of two different types of vehicles: carriers that are slow with a maximum speed V_c and unlimited endurance and small vehicles that are faster with $V_v \geq V_c$ but have a limited endurance $a \geq 0$. Both types of vehicles can cooperate such that the carriers can deploy, recover, and service the UAVs. Such a system composed by a single carrier and a single vehicle is defined in detail in [14].

Consider a mission where the carrier-vehicle system is composed of one fast vehicle and two carriers. The mission consists of the visit by the fast vehicle of an ordered set $Q = q_1, \dots, q_n$ of target points in the 2D plane. The aim is to define the trajectories of the three vehicles such that the mission time completion is minimized.

As shown in [14], it is enough to define the position of the system at the take-off and landing points. Therefore, let us define $x_{to,i} \in \mathbb{R}^2$ as the take-off position for the target point i and $x_{l,i} \in \mathbb{R}^2$ its landing position. Regarding the carriers, it is worth to notice that $x_{to,i}$ is also the position of the carrier from which the vehicle takes off, while $y_{to,i} \in \mathbb{R}^2$ can be defined as the position of the other carrier at the exact same instant. Similarly, $x_{l,i}$ represents the position of the carrier on which the vehicle lands after visiting the i-th target points, while $y_{l,i} \in \mathbb{R}^2$ denotes the position of the other carrier. These variables allow to define the position of the whole carrier-vehicle system all along the mission.

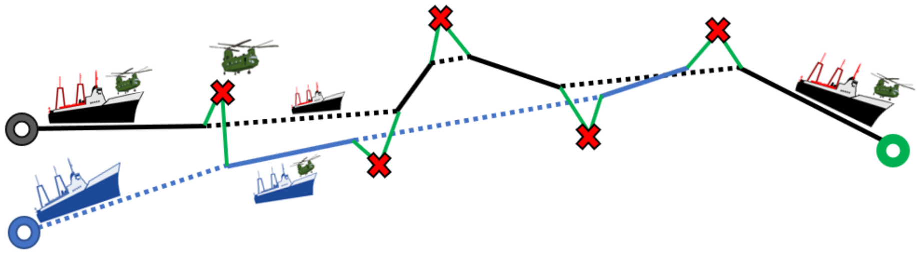

Fig. 1: Schematic of the Multiple carrier-vehicle salesman problem in a maritime scenario.

The vehicle path in the case of such a team presents two kind of time intervals. The time when the vehicle is on board of a carrier denoted by $t_i^{l,to}$, $i = 1, \dots, n$ and when the vehicle is airborne denoted by $t_i^{to,l}$,$i = 1, \dots, n+1$. Knowing that the vehicle flight time is limited by the endurance a, the following constraints must be satisfied:

$$0 \leq t_i^{to,l} \leq a \quad i = 1, \dots, n \tag{1}$$

$$0 \leq t_i^{l,to} \quad i = 1, \dots, n+1 \tag{2}$$

Each point visit is composed of two line segments: from vehicle take-off position $x_{to,i}$ to target point q_i and from target point to vehicle landing position $x_{l,i} \in \mathbb{R}^2$, with $i = 1, \dots, n$. Assuming that the elapsed time at target point is null, the following constraint must be considered:

$$\|x_{to,i} - q_i\| + \|q_i - x_{l,i}\| \leq V_v t_i^{to,l} \quad i = 1, \dots, n \tag{3}$$

Between landing and takeoff instants, the vehicle must remain on one of the carriers. Therefore, during these periods it is considered as part of the carrier system, moving with a maximum speed V_c, following that:

$$\|x_{l,i-1} - x_{to,i}\| \leq V_c t_i^{l,to} \quad i = 1, \ldots, n+1 \tag{4}$$

$$\|y_{l,i-1} - y_{to,i}\| \leq V_c t_i^{l,to} \quad i = 1, \ldots, n \tag{5}$$

where $x_{l,0} = x_{c,0}$, $y_{l,0} = y_{c,0}$ and $x_{to,n+1} = x_f$.

In the case considered in this paper, after each takeoff, the vehicle can either return to the carrier or land on the other one. This behaviour is denoted by the binary decision variable α_i with $i = 1, \ldots, n$. This variable takes the value of 1 when the vehicle is landing on the same carrier, or 0 when it switches.

This behaviour can be denoted as follows

$$\alpha_i \in \{0, 1\} \quad i = 1, \ldots, n \tag{6}$$

$$\alpha_i \|x_{to,i} - x_{l,i}\| \leq V_c t_i^{to,l} \quad i = 1, \ldots, n \tag{7}$$

$$\alpha_i \|y_{to,i} - y_{l,i}\| \leq V_c t_i^{to,l} \quad i = 1, \ldots, n \tag{8}$$

$$(1 - \alpha_i)\|x_{to,i} - y_{l,i}\| \leq V_c t_i^{to,l} \quad i = 1, \ldots, n \tag{9}$$

$$(1 - \alpha_i)\|y_{to,i} - x_{l,i}\| \leq V_c t_i^{to,l} \quad i = 1, \ldots, n \tag{10}$$

Since the goal of the mission is to minimize the total travelling time, the solution is equivalent to the minimization of the sum of all time segments corresponding to the vehicle path phases. The optimization problem can be given in the form of Mixed Integer Non-linear Programming (MINLP) problem as

$$\begin{aligned} \underset{\alpha,x,y,t}{\text{minimize}} \quad & \left(\sum_{i=1}^{n} t_i^{to,l} + \sum_{i=1}^{n+1} t_i^{l,to}\right) \\ \text{subject to} \quad & (1) - (5), (6) - (10). \end{aligned} \tag{11}$$

The non-linearity in constraints (7)-(10) makes the formulation complex to solve with current solvers. However, similar to what is done in [15], it is possible to rewrite constraints (7)-(10) as second order cone constraints, where THE equivalent logical expression

$$(\alpha_i = 1) \Rightarrow \|x_{to,i} - x_{l,i}\| \leq V_c t_i^{to,l} \tag{12}$$

can be reformulated as

$$\|x_{to,i} - x_{l,i}\| \leq V_c t_i^{to,l} + (1 - \alpha_i)L \quad i = 1, \ldots, n \quad . \tag{13}$$

Similarly, the other constraints can be rewritten as

$$\|y_{to,i} - y_{l,i}\| \leq V_c t_i^{to,l} + (1 - \alpha_i)L \quad i = 1, \ldots, n \tag{14}$$

$$\|x_{to,i} - y_{l,i}\| \leq V_c t_i^{to,l} + \alpha_i L \quad i = 1, \ldots, n \tag{15}$$

$$\|y_{to,i} - x_{l,i}\| \leq V_c t_i^{to,l} + \alpha_i L \quad i = 1, \dots, n. \tag{16}$$

Therefore, the optimization problem is now given as follows

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & \left(\sum_{i=1}^{n} t_i^{to,l} + \sum_{i=1}^{n+1} t_i^{l,to}\right) \\ \text{subject to} \quad & (1)-(5), (6), (13)-(16) \end{aligned} \tag{17}$$

being a Mixed-Integer Second Order Conic Program (MISCOP) where L is a large enough real positive number. An example of a solution for 10 points is given in Figure 2.

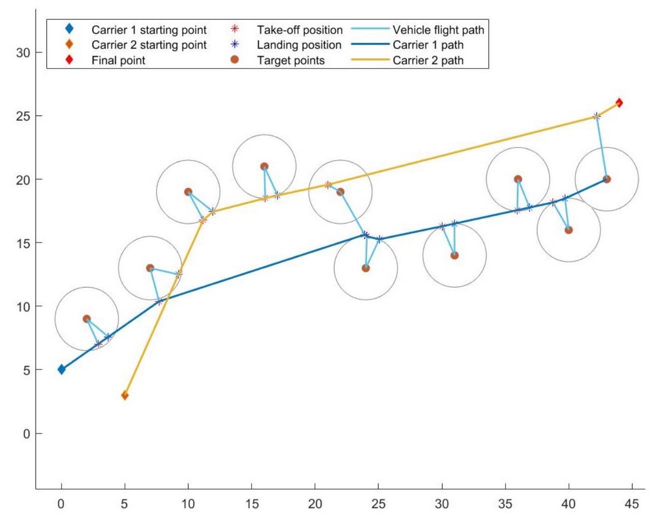

Fig. 2: Example of a mission of 10 points

The L parameter has to be large enough but a too large value would increase drastically the computational time [16]. Fortunately, this parameter has a physical meaning illustrated in Figure 3 (for the case of $\alpha_i = 0$) in terms of distance between vehicles. Therefore, given the characteristics of the problem, it is possible to analytically determine the interval of L_{min} such that the constraints (13), (14),(15) and (16) hold true. Figure 3 shows a configuration of $x_{to,i}, x_{l,i}, y_{to,i}, y_{l,i}$ and the corresponding interval of L. L_1 and L_2 are, respectively, the minimum distances such that constraints (13) and (14) are no longer active. The green point $x_{L,2}$ corresponds to the worst case scenario in which the distance $\|x_{to,i} - x_{l,i}\|$ is maximal and equal to aV_v. It corresponds to the case in which $t_i^{to,l} = a$. Similarly, the point $y_{TO,2}$ corresponds to the worst case scenario in which the distance $\|y_{to,i} - y_{l,i}\|$ is maximal and equal to $aV_v + 2aV_c$.

Therefore, the parameter L can be computed beforehand as

$$L_{min} = \max(a(V_v + V_c), \|x_{to,i} - y_{to,i}\| \quad i = 1, \dots, n) \tag{18}$$

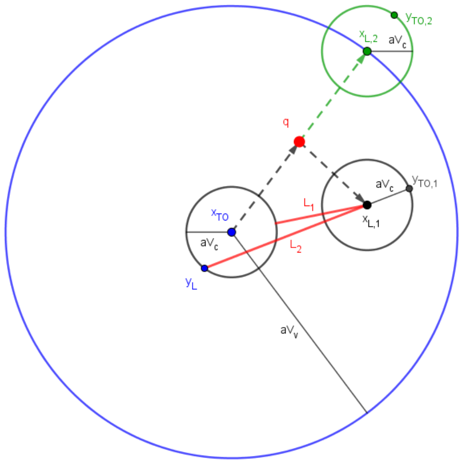

Fig. 3: Illustration of minimum distance L for the constraints (13) and (14).

being convex problem easy to solve.

3 Proposed heuristic

The mixed-integer formulation proposed in (17) provides an optimal solution, but given the NP-hard nature of the problem becomes computationally expensive for large inputs. Hence, in this section we propose a fast heuristic able to solve the problem with satisfactory results.

Given the integer relaxation of the constraint (6) as

$$0 \leq \alpha_i \leq 1 \quad i = 1, \dots, n \tag{19}$$

the resulting formulation

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & (\sum_{i=1}^{n} t_i^{to,l} + \sum_{i=1}^{n+1} t_i^{l,to}) \\ \text{subject to} \quad & (1)-(5),(13)-(16),(19) \end{aligned} \tag{20}$$

is a Second-Order Conic Programming (SOCP) problem, a kind of convex problem easy to solve. The presented heuristic takes this result as an input for a rounding algorithm. The main idea behind the heuristic lies in the fact that the solution from the relaxed problem (20) can be seen as a probability on the variable α to be chosen 1. Therefore the final trajectory calculation takes the values more likely to provide a better solution

However, since $\alpha_i \in [0, 1]$, in constraints (13)-(16) the geometric meaning of parameter L is lost. The solution and computational time becomes sensitive to the value of L. Based on an empirical approach, the following formula was obtained:

$$L_{approx} = 1.6d^*(\frac{aV_v}{d^*} + 0.068)(0.88 - \frac{V_c}{V_v}) \tag{21}$$

where d^* is the average length separating successive target points. This expression allows, depending on the configuration parameters of the problem, to select a value of L that very likely provides a close to optimal solution. Being a first tentative to define an appropriate value of L, the heuristic considers a range of values of L with 20% and 40% deviation to avoid unexpected results.

The main steps of the heuristic are schematically represented as follows:

Given : $X = \{q,\ x_{0,c},\ y_{0,c},\ x_f,\ a,\ V_v,\ V_c\}$
Find : $x_{to,i}\ y_{to,i}\ x_{l,i}\ y_{l,i}\ \alpha$

Compute L_{approx} with eq. (21)
$L_{list} = L_{approx} \times [0.6, 0.8, 1, 1.2, 1.4]$
For each L_i in L_{list}
   Solve relaxed SOCP problem (20)
   get α
   round α
   Solve original problem (11) with rounded α
   get f_i
end
$f_{heuristic} = min(\{f_i, :\ i = 1, \ldots, 5\})$

4 Numerical results

This section shows different numerical simulations in terms of computational time and optimality to compare the performance of the mixed-integer formulation and the proposed heuristic. A series of randomized simulations have been performed to compare the results with different points distributions. The simulations have been using MOSEK v9.0.89 for the convex problems and GUROBI solver for the mixed-integer problems, both in YALMIP environment.

Figure 4 represents the level of degradation Δ of the heuristic solution related to the optimal solution among 500 simulations. Due to the computing time of the mixed-integer program the simulations considered only a set of 9 visit points.

In Table 1 and Table 2 the results from these simulations are detailed. It can be seen how the average degradation of the solution respect to the optimal value is only 2.47% and that 82% of the cases have less that 5% degradation.

A second comparison in terms of computational time was also carried out. Figure 5 represents the evolution of computational times with respect to the number of target points. The results show that the heuristic approach grows linearly with the number of target points meanwhile the mixed-integer solution drastically increase for more than 16 points.

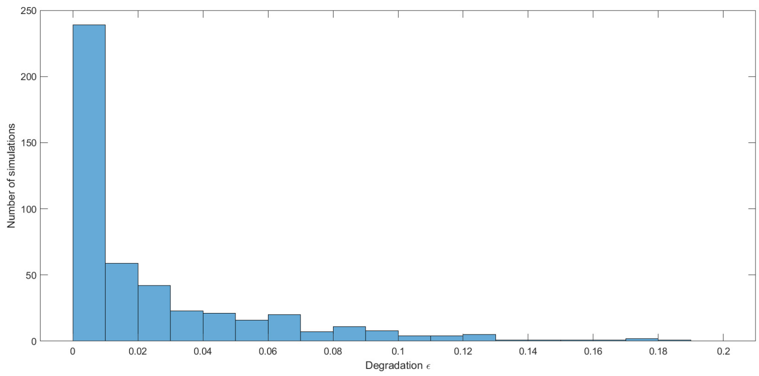

Fig. 4: Distribution of the level of degradation Δ in the simulations.

N.of points	Avg deg.	Max deg.
9	2.47%	28.15%

Table 1: Degradation of the heuristic result respect to the optimal.

N. of points	$< 1\%$	$< 5\%$	$< 10\%$	$< 15\%$
9	53.8%	82.80%	95.20%	98.2%

Table 2: Results distribution regarding different degradation thresholds.

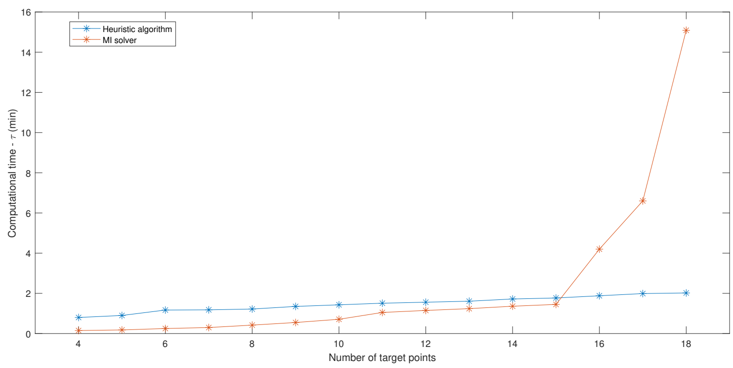

Fig. 5: Comparison of computational time evolution with respect to the number of target points.

These results support the clear advantage of the proposed heuristic when the number of points becomes larger. Additionally, it shows the good performance in terms of optimality.

5 Conclusion

This paper introduces a novel extension of the Carrier-Vehicle Travelling Salesman Problem considering the use of two carriers. The presented results show the advantages of the considered scenario and a proper way to formulate and solve the problem. The problem and constraints are defined in a way that allows to write it as Mixed-Integer Second Order Program (MISOCP) which provides optimal results. Additionally, a simple and fast heuristic is also defined, which is proved to have good results in terms of optimality and computational time.

The obtained results can be used for the planning of monitoring activities or similar logistic problems involving a team of two carriers and one vehicle. Future extensions could allow the faster vehicle to visit multiple points with a single takeoff.

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