A Distributed Swarm Aggregation Algorithm for Bar Shaped Multi-Agent Systems

Renzo F. Carpio, Letizia Di Giulio, Emanuele Garone, Giovanni Ulivi and Andrea Gasparri

Abstract— **In this work we consider a swarm of agents shaped as bars with a certain orientation in the state space. Members of the swarm have to reach an aggregate state, while guaranteeing the collision avoidance and possibly achieving an angular consensus. By relying on a segment-to-segment distance definition, we propose a control law, which guides the agents towards this goal. A theoretical analysis of the proposed control scheme along with simulations and experimental results is provided. The proposed framework can be used to model several application scenarios ranging from collaborative transportation to precision farming, where each agent may represent either a large robot or a group of robots intent to carry bar-like shaped loads. Representative examples include: a fleet of robot-teams performing a collaborative object transportation task in an automated logistic setting, or a fleet of autonomous tractors each carrying a large atomizer to spray chemical products for pest and disease control in a precision farming setting.**

I. INTRODUCTION

In the last two decades, distributed multi-agent coordination has been a very attractive research field in the robotics and control communities [1], [2], [3]. This topic has gained momentum as multi-agent systems offer many advantages ranging from robustness and reliability to reduced complexity of hardware design for the agents. A very common approach for developing coordination algorithm is to take inspiration from biology. For example, several multi-agent swarming protocols have been inspired by animal aggregations such as schools of fish, flocks of birds or swarms of bees, that are believed to use simple, local motion coordination rules at the individual level, [4], [5], [6]. As a matter of fact, swarming models have been a good solution to many engineering and computer science problems. One of the first related works is [7], where the authors describe and simulate a flock of birds that fly following a swarming model based on few simple rules and local interactions. In [8] it is possible to find a focus on ideas and concepts for the advancement of swarm robotics as an engineering field. The authors describe, from a general point of view, the limits, the advantages and the future developing of this discipline. In [9], the authors show a theoretical explanation for the behavior observed by Vicsek. In addition, convergence results are derived for several other similarly inspired models. For example, in [10] is considered an asynchronous distributed control for geometric pattern formation of multiple anonymous agents. A proof for the stability of the formation can be found in [11], [12], where the authors use a control Lyapunov function and formation constrains. Another Lyapunov function is used in [13] to show the convergence of the system to a steady state in a desired area, in which the agents move while preserving a minimal inter-distance. Many of the cited articles consider decentralized formation control laws. An example of this type of control is the one in [14], where is developed a decentralized controller to generate a desired two-dimensional geometric pattern for a swarm. One of the works that has brought to the swarming research community significant results, is the one by Gazi and Passino in [15], where a decentralized aggregation algorithm based on a continuous-time control law is given. Several papers have been successively developed along this direction, such as [16], [17], [18].

In this work, we focus on the swarming problem, that is the problem of reaching an aggregate state, while guaranteeing collision avoidance and possibly achieving an angular consensus for a swarm of agents, which are shaped as bars with a certain orientation in the state space. The proposed framework can be used to model several application scenarios ranging from collaborative transportation to precision farming, where each agent may represent either a large robot or a group of robots intent to carry bar-like shaped loads. Notably, swarm robotics has been recently recognized as a promising research direction in the context of precision farming, e.g. [19], [20]. As a representative example for our setting, one may think to a fleet of autonomous tractors each carrying a large atomizer in order to spray chemical products on the canopy of trees for a pest and disease control setting.

A work which is similar in spirit to ours is [21], where the authors focus on the formation control and obstacle avoidance of multiple rectangular agents with limited communication ranges. Compared to [21], in our work no predefined shape is considered, and in line with typical swarming models, the aggregative behavior just emerges from the local interaction among the agents.

II. PRELIMINARIES

A. Multi-Agent and Network Modeling

Let us consider a swarm composed by N agents in a 2-dimensional Euclidean space, where each individual is shaped as a bar (line segment). The state vector $\mathbf{s}$ of the system is composed by the state s_i of each agent v_i in the swarm as $\mathbf{s} = \begin{bmatrix} s_1^T & \dots & s_N^T \end{bmatrix}$, $s_i = \begin{bmatrix} p_i^T & \theta_i \end{bmatrix}^T$ where $p_i = [p_{i,1}\, p_{i,2}]^T \in R^2$ represents the position of the middle point of the bar and θ_i represents its orientation with

R. F. Carpio, G. Ulivi and A. Gasparri are with the Department of Engineering, University of “Roma Tre”, Via della Vasca Navale 79, 00146 Rome, Italy.

L. Di Giulio is with the Safety Office of the Experimental Department at CERN, 1211 Geneva, Switzerland.

E. Garone is with the Faculty of Applied Science, Control and Systems Analysis Dep., Université Libre de Bruxelles, 1050 Brussels, Belgium

This work has been supported by the European Commission under the grant agreement number 774571 – Project PANTHEON.

Corresponding Author: A. Gasparri – E-mail: *gasparri@dia.uniroma3.it*

regards to the x (horizontal) axis. Note that, for each bar-shaped agent v_i the orientation $\theta_i \in (-\pi/2, \pi/2]$ is computed as the minimum angle of the bar with respect to the x axis, with positive value when considering counter-clockwise rotations. Furthermore, for each agent v_i the length of the bar is assumed to be $2h$. Let us now denote the network topology encoding the agent-to-agent interactions with an undirected graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$, where $\mathcal{V} = \{v_i\}$, with $i \in \{1, \ldots, N\}$ represents the set of agents in the swarm and $\mathcal{E} = \{e_{ij}\}$ the set of arcs e_{ij} that links every pair of interacting individuals v_i and v_j. Let us denote with $\mathcal{N}_i$ the neighborhood of a bar-shaped agent v_i, that is the set of agents v_j for which $e_{ij} \in \mathcal{E}$. Let us denote with the matrix $\mathcal{A}$ the adjacency matrix of the graph $\mathcal{G}$ where an element $a_{ij} = 1$ if there is an edge $e_{ij} \in \mathcal{E}$ between two agents v_i and v_j, and $a_{ij} = 0$ otherwise. Note that since the graph $\mathcal{G}$ is undirected, it follows that $a_{ij} = a_{ji}$. In addition, let us denote with $\mathcal{D}$ the degree matrix of the graph $\mathcal{G}$, which is a diagonal matrix defined as $\mathcal{D} = \mathrm{diag}\{|\mathcal{N}_i|\}$. Finally, let us denote with $\mathcal{L}$ the Laplacian matrix defined as $\mathcal{L} = \mathcal{D} - \mathcal{A}$.

B. Segment to Segment Distance Modeling

We now briefly review, the segment-to-segment modeling given in [22]. Considering two sets A and B, the minimum distance between them can be formally defined as

$$d(A, B) = inf_{x 2 A,\, y 2 B}\, ||x - y||, \tag{1}$$

where $\|\cdot\|$ denotes the Euclidean norm. Consider two endpoints p_i^A and p_i^B in a 2-dimensional Euclidean space with coordinates $p_i^A = [p_{i,1}^A \quad p_{i,2}^A]^T$ and $p_i^B = [p_{i,1}^B \quad p_{i,2}^B]^T$ The line-segment $\overline{p_i^A\, p_i^B}$ passing through these two points can be defined as $p_i(t_i) = p_i^A(1-t_i) + p_i^B\, t_i$ where $t_i \in [0, 1]$. The endpoints of the segment correspond to the values 0 and 1. To compute the distance between two segments the first step is to compute the intersection point between the two lines on which the segments lie. For non-parallel lines the parameters t_i and t_j where this happens are

$$\begin{aligned} t_{ij}^{\min} &= \frac{\sum_{k=1}^{2} d_{i,k}\, d_{ij,k} \sum_{k=1}^{2} d_{j,k}^2 - \sum_{k=1}^{2} d_{j,k}\, d_{ij,k} \sum_{i=1}^{2} d_{i,k}\, d_{j,k}}{\sum_{k=1}^{2} d_{i,k}^2 \sum_{k=1}^{2} d_{j,k}^2 - \left(\sum_{k=1}^{2} d_{i,k} d_{j,k}\right)^2} \\ t_{ji}^{\min} &= -\frac{\sum_{k=1}^{2} d_{j,k}\, d_{ij,k} \sum_{k=1}^{2} d_{i,k}^2 - \sum_{k=1}^{2} d_{i,k}\, d_{ij,k} \sum_{k=1}^{2} d_{i,k} d_{j,k}}{\sum_{k=1}^{2} d_{i,k}^2 \sum_{k=1}^{2} d_{j,k}^2 - \left(\sum_{k=1}^{2} d_{i,k}\, d_{j,k}\right)^2} \end{aligned} \tag{2}$$

where

$$d_{i,k} = p_{i,k}^B - p_{i,k}^A,\ d_{j,k} = p_{j,k}^B - p_{j,k}^A,\ d_{ij,k} = p_{j,k}^A - p_{i,k}^A \tag{3}$$

These $t_{ij}^{\min}$ and $t_{ji}^{\min}$ correspond to some points on the lines passing through the points $\{p_i^A, p_i^B\}$ and $\{p_j^A, p_j^B\}$. They may or may not lie within the segments $\overline{p_i^A\, p_i^B}$ and $\overline{p_j^A\, p_j^B}$. If both lie in the segment it means that the segments intersect, and thus $d_{ij} = 0$. In the case when at most one of the two points lies within the segment, according to [22], the following holds true:

***Lemma** 1:* [22] The minimum distance between two segments l_i and l_j is calculated by first computing the points of minimum distance $\{t_{ij}^{\min}, t_{ji}^{\min}\}$ and then taking the closest points $\{\bar{t}_{ij}, \bar{t}_{ji}\}$ to these points within the segments, that is

$$\begin{aligned} \bar{t}_{ij} &= \underset{t_{ij}\ 2\,[0,\,1]}{\arg\min} \left\{ \left\| p_i(t_{ij}^{\min}) - p_i(t_{ij}) \right\| \right\} \\ \bar{t}_{ji} &= \underset{t_{ji}\ 2\,[0,\,1]}{\arg\min} \left\{ \left\| p_j(t_{ji}^{\min}) - p_j(t_{ji}) \right\| \right\} \end{aligned} \tag{4}$$

In general, the actual distance d_{ij} between the two segments $\overline{p_i^A\, p_i^B}$ and $\overline{p_j^A\, p_j^B}$ can be computed as follows

$$d_{ij} = \sqrt{\sum_{k=1}^{n} \left(d_{i,k}\, \bar{t}_{ij} - d_{j,k}\, \bar{t}_{ji} - d_{ij,k}\right)^2}. \tag{5}$$

Note that, (2) gives a indeterminate form $0/0$ in the case of parallel segments. As pointed out in [22], the points of minimum distance can be still derived by choosing $t_{ij}^{\min} = 0$ and deriving $t_{ji}^{\min}$ accordingly as

$$t_{ji}^{\min} = -\left(\sum_{k=1}^{n} d_{j,k}\, d_{ij,k}\right) \Big/ \left(\sum_{k=1}^{n} d_{j,k}^2\right) \tag{6}$$

The reader is referred to [22] for a comprehensive overview of the segment-to-segment distance modeling.

III. Distance Formulae for Bar Shaped Multi-Agent Systems

In our framework, the formula of the distance given in eq. (5) can be more conveniently re-written in terms of the agents middle-points and orientation[1]. Let us consider two bar-shaped agents v_i and v_j with states $s_i = [p_i^T\ \theta_i]^T$ and $s_j = [p_j^T\ \theta_j]^T$, respectively. For an agent v_i oriented parallel to the x axis, that is $\theta_i = 0$, let us denote with p_i^A and p_i^B the endpoint on the right and left of the segment middle-point, respectively. With that in mind, these endpoints can be expressed as follows

$$p_i^A = p_i - h[\cos(\theta_i)\ \sin(\theta_i)]^T, \quad p_i^B = p_i + h[\cos(\theta_i)\ \sin(\theta_i)]^T \tag{7}$$

Thus, the distance in eq. (5) can be re-written as

$$d_{ij} = \sqrt{\begin{aligned}&(p_{i,1} - p_{j,1} + \alpha\cos(\theta_i) + \beta\cos(\theta_j))^2 + \\ &+(p_{i,2} - p_{j,2} + \alpha\sin(\theta_i) + \beta\sin(\theta_j))^2\end{aligned}} \tag{8}$$

with $\alpha = h(-1 + 2t_{ij})$ and $\beta = h(1 - 2t_{ji})$.

In addition, the closed form expression of the distance gradient with respect to the states s_i and s_j two bar-shaped agents is

$$\nabla_{\mathbf{s}} d_{ij} = \left[0_3^T\ \ldots\ \nabla_{s_i} d_{ij}^T\ \ldots\ \nabla_{s_j} d_{ij}^T\ \ldots\ 0_3^T\right]^T \tag{9}$$

where 0_3 denotes a 3-dimensional vector of zeros and the term $\nabla_{s_i} d_{ij}$ and $\nabla_{s_j} d_{ij}$ are defined as

$$\begin{aligned} \nabla_{s_i} d_{ij} &= \left[\nabla_{p_i} d_{ij}^T\ \nabla_{\theta_i} d_{ij}\right]^T = \left[\nabla_{p_{i;\,1}} d_{ij}\ \nabla_{p_{i;\,2}} d_{ij}\ \nabla_{\theta_i} d_{ij}\right]^T \\ \nabla_{s_j} d_{ij} &= \left[\nabla_{p_j} d_{ij}^T\ \nabla_{\theta_j} d_{ij}\right]^T = \left[\nabla_{p_{j;\,1}} d_{ij} \nabla_{p_{j;\,2}} d_{ij}\ \nabla_{\theta_j} d_{ij}\right]^T \end{aligned} \tag{10}$$

[1]We reiterate that in this work we focus on controlling the middle point and orientation of the bar-shaped agents for the sake of coordination. As it will be shown in Section V, in case two robots determine the end-points of the segment, a proper low-level control law should be provided accordingly.

with

$$\begin{bmatrix} \nabla_{p_{i;\,1}} d_{ij} \\ \nabla_{p_{i;\,2}} d_{ij} \\ \nabla_{\theta_i} d_{ij} \end{bmatrix} = \frac{1}{d_{i\,j}} \begin{bmatrix} p_{ij,1} + c_{ij} + c_{ji} \\ p_{ij,2} + s_{ij} + s_{ji} \\ -s_{ij}(p_{ij,1} + c_{ij} + c_{ji}) + c_{ij}(p_{ij,2} + s_{ij} + s_{ji}) \end{bmatrix} \tag{11}$$

and

$$\begin{bmatrix} \nabla_{p_{j;\,1}} d_{ij} \\ \nabla_{p_{j;\,2}} d_{ij} \\ \nabla_{\theta_j} d_{ij} \end{bmatrix} = \frac{1}{d_{i\,j}} \begin{bmatrix} -(p_{ij,1} + c_{ij} + c_{ji}) \\ -(p_{ij,2} + s_{ij} + s_{ji}) \\ -s_{ji}(p_{ij,1} + c_{ij} + c_{ji}) + c_{ji}(p_{ij,2} + s_{ij} + s_{ji}) \end{bmatrix} \tag{12}$$

with $p_{ij,1} = (p_{i,1} - p_{j,1})$, $p_{ij,2} = (p_{i,2} - p_{j,2})$, $c_{ij} = \alpha \cos(\theta_i)$, $c_{ji} = \beta \cos(\theta_j)$, $s_{ij} = \alpha \sin(\theta_i)$, and $s_{ji} = \beta \sin(\theta_j)$.

A. *Segment to Segment Distance Properties*

The minimum distance formula that we use in this work is a continuous function. It is differentiable in the whole domain, except in $d_{ij} = 0$, where the function has a point of non differentiability. Although a more rigorous analysis would require the usage of tools coming from the non-smooth analysis along with the concept of the Clarke generalized gradient [23], in the following we limit our-self to the classical Lyapunov analysis by assuming (and showing) that the system evolution does not pass through such a point of discontinuity, i.e., a configuration for which $d_{ij} = 0$, for any $e_{ij} \in \mathcal{E}$. In fact, as we will prove in the Section IV-A, the control law is designed in such a way to ensure that the minimum distance between two agents is always kept greater then zero.

We now list a set of properties for the minimum distance formula given in (8) that will be used in the following. Proofs of these properties are here omitted due to space limitations.

1) ***Property 1:*** $\nabla_{p_i} d_{ij} = -\nabla_{p_j} d_{ij}$

2) ***Property 2:*** $d_{ij} \leq \|p_i - p_j\|$

3) ***Property 3:*** $(p_i - p_j)^T \nabla_{p_i} d_{ij} \geq 0$

IV. Swarm Aggregation Control Law Design

We now introduce a control law by which the bar-shaped multi-agent system can reach an aggregate state, while guaranteeing the collision avoidance and possibly achieving an angular consensus. To achieve this objective, we resort to the machinery of potential-based control design, a very popular framework widely exploited in the robotics and control communities for controlling multi-agent systems, [24], [25]. In particular, each agent v_i to control the middle point p_i and the orientation θ_i runs the following distributed (local) control law[2]

$$\begin{aligned} \dot{p}_i &= -\sum_{j\,2\,N_i} \nabla_{p_i} d_{ij} \, \left(a d_{ij} - b/d_{ij}^2\right) \\ \dot{\theta}_i &= -\sum_{j\,2\,N_i} \nabla_{\theta_i} d_{ij} \left(a d_{ij} - b/d_{ij}^2\right) - \sum_{j\,2\,N_i} \sin\left(\theta_j - \theta_i\right) \end{aligned} \tag{13}$$

with d_{ij} the segment to segment minimum distance between the bar-shaped agents v_i and v_j as defined in Section III.

[2]It should be noticed that several potentials could be used to derive suitable pairwise aggregative and repulsive forces for the agents. Here, for the sake of simplicity and with no lack of generality the potentials proposed in [26] are used.

The first equation, $\dot{p}_i$, is the translational dynamics of the middle-point of agent v_i, and guarantees an aggregative behavior as in [26], but using the segment-to-segment minimum distance instead of the Euclidean distance. The second equation, $\dot{\theta}_i$, is the rotational dynamics of an agent v_i, and guarantees an aggregative behavior through the first term, which similarly to the translational dynamics is based on the segment-to-segment minimum distance, and possibly the achievement of an angular consensus through the second term, which follows from the N-torus synchronization protocol described in [27]

A. *Properties Analysis*

Let us now characterize the swarming behavior of the multi-agent system where each bar-shaped agent v_i runs the control law given in eq. (13). In particular, let us prove that the multi-agent system exhibits an aggregative behavior and no collision occurs.

Theorem 1: Consider a multi-agent system where each bar-shaped agent v_i is running the control law given in eq. (13) and assume that the initial state $\mathbf{s}(0)$ of the system is such that agents do not overlap, that is $d_{ij}(0) \neq 0$, $\forall i, j \in \mathcal{V}$. Then the multi-agent system reaches an aggregate state, while guaranteeing the collision avoidance over time and possibly achieving an angular consensus.

Proof: Consider the following Lyapunov function, which is related to the distance and orientation between the bar-shaped agents:

$$J(\mathbf{s}) = \underbrace{\sum_{i=1}^{N\;1} \sum_{j\,2\,N_i\,,\, j>i} \left[\frac{a\, d_{ij}^2}{2} + \frac{b}{d_{ij}} \right]}_{J_1(\mathbf{s})} + \underbrace{\langle e^{j\,\theta\,T},\, \mathcal{L}\, e^{j\,\theta} \rangle}_{J_2(\mathbf{s})} \tag{14}$$

with $e^{j\,\theta} = [e^{j\,\theta_1}, \ldots, e^{j\,\theta_N}]^T$, where $e^{j\theta_i}$ denotes a phasor with phase θ_i, and $\langle z_1, z_2 \rangle = \mathsf{Re}\{\bar{z}_1\, z_2\}$. Note that critical points of the term $J_1(\mathbf{s})$ encodes multi-agent configurations corresponding to an aggregative behavior, while critical points of the term $J_2(\mathbf{s})$ encodes multi-agent configurations corresponding to angular alignments of the bars. By taking the time derivative of the first potential $J_1(\mathbf{s})$, we obtain

$$\dot{J}_1(\mathbf{s}) = (\nabla_{\mathbf{s}} J_1)^T \, \dot{\mathbf{s}} = \sum_{i=1}^{N} \left[(\nabla_{p_i} J_1)^T \, \dot{p}_i + (\nabla_{\theta_i} J_1)^T \, \dot{\theta}_i \right], \tag{15}$$

where $\nabla_{p_i} J_1$ is defined as follows

$$\nabla_{p_i} J_1 = \sum_{j\,2\,N_i} \nabla_{p_i} d_{ij} \, (a d_{ij} - b/d_{ij}^2) = -\dot{p}_i \tag{16}$$

and $\nabla_{\theta_i} J_1$ is defined as follows

$$\nabla_{\theta_i} J_1 = \sum_{j\,2\,N_i} \nabla_{\theta_i} d_{ij} \, (a d_{ij} - b/d_{ij}^2) \tag{17}$$

Furthermore, by taking the time derivative of the second potential $J_2(\mathbf{s})$, we obtain

$$\dot{J}_2(\mathbf{s}) = (\nabla_{\mathbf{s}} J_2)^T \, \dot{\mathbf{s}} = \sum_{i=1}^{N} \left[(\nabla_{p_i} J_2)^T \, \dot{p}_i + (\nabla_{\theta_i} J_2)^T \, \dot{\theta}_i \right], \tag{18}$$

where $\nabla_{p_i} J_2 = 0^T$ by construction and $\nabla_{\theta_i} J_2$ is defined as follows

$$\nabla_{\theta_i} J_2 = \sum_{j\in\mathcal{N}_i} \sin(\theta_j - \theta_i) \qquad (19)$$

At this point, plugging together eqs. (15) and (18), and by using respectively eqs. (16)-(17) and eq. (19), it follows

$$\begin{aligned}\dot J(s) &= \dot J_1(s) + \dot J_2(s)\\ &= \sum_{i=1}^{N} \underbrace{\left[(\nabla_{p_i} J_1)^T \dot p_i + (\nabla_{\theta_i} J_1 + \nabla_{\theta_i} J_2)^T \dot\theta_i\right]}_{\dot J_i}\end{aligned} \qquad (20)$$

Let us now analyze each term $\dot J_i$ by plugging the control law given in eq. (13) as follows

$$\begin{aligned}\dot J_i &= (\nabla_{p_i} J_1)^T \dot p_i + (\nabla_{\theta_i} J_1 + \nabla_{\theta_i} J_2)^T \dot\theta_i\\ &= -k\|\nabla_{p_i} J_1\|^2 - k\|\nabla_{\theta_i} J_1 + \nabla_{\theta_i} J_2\|^2\end{aligned} \qquad (21)$$

By exploiting (21), let us now rewrite the (collective) Lyapunov derivative as follows

$$\begin{aligned}\dot J(s) &= \sum_{i=1}^{N} -k\|\nabla_{p_i} J_1\|^2 - k\|\nabla_{\theta_i} J_1 + \nabla_{\theta_i} J_2\|^2\\ &= \sum_{i=1}^{N} \left[-k\|\dot p_i\|^2 - k\|\dot\theta_i\|^2\right]\end{aligned} \qquad (22)$$

which demonstrates that by construction the Lyapunov derivative is negative semidefinite. At this point, to demonstrate that no collision occurs over time it suffices to notice that if a certain distance $d_{ij}(t)$ between two agents v_i and v_j were to go to zero over time, i.e., $d_{ij}(t) \to 0$, then by construction the Lyapunov function would go to infinity, i.e., $J(s) \to \infty$. However, since the Lyapunov derivative is negative semidefinite, the Lyapunov function cannot increase over time, and thus no collision can occur. Furthermore, being the Lyapunov derivative negative semidefinite it follows that a critical point of the potential function $J(s)$ is eventually reached, thus the multi-agent system exhibits an aggregative behavior, while guaranteeing the collision avoidance over time and possibly achieving an angular consensus. ∎

B. Cohesiveness Analysis

Let us now show that the the distance between any pair of agents constituting the multi-agent system will eventually be upper bounded as time goes to infinity.

Theorem 2: Consider a multi-agent system where each bar-shaped agent v_i is running the control law given in eq. (13) and assume that the initial state $s(0)$ of the system is such that agents do not overlap, that is $d_{ij}(0) \neq 0;\ \forall i,j \in \mathcal{V}$. Then as $t \to \infty$ we have that $p(t) \in \mathcal{B}_e$, with

$$\mathcal{B}_e = \left\{ p \in \mathbb{R}^2 : d_{ij} \le N\left(\sqrt[3]{\frac{b}{a}} + 2h\right);\ \forall i,j \in \mathcal{V} \right\} \qquad (23)$$

Proof: In order to prove the theorem, let us assume, for the sake of simplicity and with no lack of generality, that the centroid of the system is $\bar p = 0_2$. Then, let us consider the following Lyapunov function

$$V(s) = \frac{1}{2}\|p\|^2 = \sum_{i=1}^{N} \|p_i\|^2 = \sum_{i=1}^{N} V_i \qquad (24)$$

for which the following derivative holds

$$\begin{aligned}\dot V(s) &= [\nabla_s V]^T \dot s = [\nabla_p V]^T \dot p = p^T \dot p\\ &= \sum_{i=1}^{N} p_i^T \dot p_i = \sum_{i=1}^{N} \dot V_i\end{aligned} \qquad (25)$$

where the last equation follows from the fact that by construction $\nabla_\theta V = 0$. Let us now detail each term $\dot V_i$ as follows

$$\dot V_i = -p_i^T \sum_{j\in\mathcal{N}_i} \nabla_{p_i} d_{ij}\left(a d_{ij} - b/d_{ij}^2\right) \qquad (26)$$

By collecting and recalling that $\nabla_{p_i} d_{ij} = -\nabla_{p_j} d_{ij}$, we have

$$\begin{aligned}\dot V(p) &= \sum_{i=1}^{N} \dot V_i = \sum_{i=1}^{N} \left(-p_i^T \sum_{j\in\mathcal{N}_i} \nabla_{p_i} d_{ij}\left(a d_{ij} - b/d_{ij}^2\right)\right)\\ &= -\sum_{e_{ij}\in\mathcal{E}} (p_i - p_j)^T \nabla_{p_i} d_{ij}\left(a d_{ij} - b/d_{ij}^2\right)\end{aligned} \qquad (27)$$

At this point, by noticing that $(p_i - p_j)^T \nabla_{p_i} d_{ij} > 0$; it follows that the Lyapunov derivative is negative definite as long as $d_{ij} > \sqrt[3]{\frac{b}{a}};\ \forall e_{ij} \in \mathcal{E}$. Therefore, it follows that, in the worst case scenario, as t goes to infinity, for any two bar-shaped robots i and j, the distance is upper bounded by

$$d_{ij} \le N\left(\sqrt[3]{\frac{b}{a}} + 2h\right); \qquad (28)$$

with $2h$ the actual length of the bar. ∎

Note that this result, even though quite conservative, is very reasonable as it describes all possible equilibria, such as for instance the case in which the multi-agent system achieve a line-shaped topology, for which the bound still holds.

V. Numerical and Experimental Validation

In this section, simulations along with experimental results are provided to corroborate the theoretical findings and the effectiveness of the proposed control law. The reader can refer to the media for a video describing the simulation and experimental results discussed in the following.

A. Low-level control for bar-shaped agents

In our setting, we assume that each bar-shaped agent v_i is composed by two robots with unicycle kinematics located at the two endpoints p_i^A and p_i^B. In particular, from eqs. (7) and (13), the desired trajectories for the two endpoints are obtained as

$$\dot p^X_{i,des} = k_1 \dot p^X_i + k_2\left(p^X_{i,des} - p^X_i\right) \qquad (29)$$

where $\dot p^X_i$ with $X \in \{A, B\}$ is obtained by deriving eq. (7), $p^X_{i,des}$ represents the desired location of one of the two endpoints, and p^X_i its actual location.

At this point, to let each pair of robots describing a bar-shaped agent track their desired trajectories we apply the well-known input/output linearization approach to unicycles [28], from which the following control law follows

$$\begin{bmatrix} q_i^X \\ w_i^X \end{bmatrix} = T(\ _{i;X})\, p_{i;des}^X \qquad (30)$$

where q_i^X and w_i^X are the linear and angular velocity of the unycicle kinematics respectively, with $X \in \{A; B\}$ and the matrix $T(\ _{i;X})$ defined as:

$$T(\ _{i;X}) = \begin{bmatrix} \cos(\ _{i;X}) & \sin(\ _{i;X}) \\ 1=(g \sin(\ _{i;X})) & 1=(g \cos(\ _{i;X})) \end{bmatrix} \qquad (31)$$

where the parameter g is the distance from the contact point of the wheel with the ground and the control point p_i^X with $X \in \{A; B\}$ located along the sagittal axis of the unicycle. It can be proven that such a control term ensures the asymptotic tracking of the desired trajectory for the control point p_i^X with $X \in \{A; B\}$, see [28] for further details.

B. Simulations

Simulations have been carried out within the ROS environment by exploiting a control framework developed by the authors in MATLAB interacting with the physical engine Gazebo [29]. In particular, a Gazebo model of the SAETTA robotic platform developed at the Robotic Lab of the Roma Tre University, which will be used for the experimental validation, has been created for the simulation. Briefly, this model encodes the main features of the SEATTA robotic platform, e.g., the unicycle kinematics, size and mass. Clearly, this allows for a fast prototyping of coordination algorithms to be run onboard the SAETTA robotic platforms. For the simulations, we have considered 20 robotic platforms to form 10 bar-shaped agents of length $2h = 0{:}665m$, and $g = 0{:}051m$ as the distance from the contact point of the wheel with the ground and the control point. Figure 1 provides performance indicators for an execution of the proposed algorithm in the simulation setting described above. In particular, Fig. 1a depicts the Lyapunov function $J(s)$ given in eq. (14) along with its derivative $\dot{J}(s)$ and it can be noticed that according to Theorem 1, $J(s)$ decreases over time being the derivative $\dot{J}(s)$ negative semidefinite by construction. Fig. 1b depicts the relative (segment-to-segment) distance between the bar-shaped agents and it can be noticed that according to Theorem 2, the relative distance is bounded over time. Finally, Fig. 1c depicts the intra-group robot-to-robot (euclidean) distance between the endpoints constituting each bar-shaped agents, and it can be noticed that as discussed in Section V-A the relative distance between pairs of robots asymptotically tracks the desired relative distance of $2h$ with a maximum deviation of $0{:}01m$, i.e., $1{:}5\%$. Note that, this information has been take into account in designing the backlash between the coupling of the two robotic platforms by means of the wood-made bar.

C. Experiments

Experiments have been carried out within the ROS environment by exploiting 6 robotic platforms SAETTA to form 3 (wood-made) bar-shaped agents of length $2h = 0{:}665m$, and $g = 0{:}051m$ as the distance from the contact point of the wheel with the ground and the control point. Briefly, the SAETTA mobile robot is a small low-cost robotic platform which features a complete sensorial system, a very accurate traction in indoor environment, and a wireless communication channel for multi-robot applications. Further details regarding the SAETTA platform can be found in [30]. Odometry information was provided by an Optitrack-based motion tracking system integrated within the ROS environment. Figure 1 provides performance indicators for an execution of the proposed algorithm for the experimental setting described above. In particular, Fig. 2a depicts the Lyapunov function $J(s)$ given in eq. (14) along with its derivative $\dot{J}(s)$ and it can be noticed again that according to Theorem 1, $J(s)$ decreases over time being the derivative $\dot{J}(s)$ negative semidefinite by construction. Fig. 2b depicts the relative (segment-to-segment) distance between the bar-shaped agents and it can be noticed that according to Theorem 2, the relative distance is bounded over time. Finally, Fig. 2c depicts the intra-group robot-to-robot (euclidean) distance between the endpoints constituting each bar-shaped agents, and it can be noticed that similarly to the simulation results also for the experimental setting the relative distance between pairs of robots asymptotically tracks the desired relative distance of $2h$ with a maximum deviation of $0{:}01m$, i.e., $1{:}5\%$.

VI. Conclusions

In this work, an aggregation problem for bar-shaped multi-agent systems has been addressed. This framework can be used to model several application scenarios where each agent may represent either a large robot or a group of robots intent to carry bar-like shaped objects. The example motivating this research was the one of a fleet of autonomous tractors each carrying a large atomizer to spray chemical products on the canopy of trees for pest and disease control in a precision farming setting. We proposed a distributed control law to the multi-agent system reach an aggregate state, while guaranteeing the collision avoidance and possibly achieving an angular consensus. A theoretical analysis of the proposed control along with simulations and experimental results has been provided. Future work will be mainly focused on investigating a scenario with time-varying interactions along with an extension for a fleet of robots with passive trailers.

References

[1] P. Yang, R. A. Freeman, and K. M. Lynch, "Multi-agent coordination by decentralized estimation and control," *IEEE Transactions on Automatic Control*, vol. 53, no. 11, pp. 2480–2496, Dec 2008.

[2] W. Ren and Y. Cao, *Distributed Coordination of Multi-agent Networks: Emergent Problems, Models, and Issues*. Springer Publishing Company, Incorporated, 2013.

[3] R. K. Williams, A. Gasparri, G. Ulivi, and G. S. Sukhatme, "Generalized topology control for nonholonomic teams with discontinuous interactions," *IEEE Transactions on Robotics*, vol. 33, no. 4, pp. 994–1001, Aug 2017.

[4] T. Vicsek, A. Czirók, E. Ben-Jacob, I. Cohen, and O. Shochet, "Novel type of phase transition in a system of self-driven particles," *Phys. Rev. Lett.*, vol. 75, pp. 1226–1229, Aug 1995.

[5] A. Okubo, "Dynamical aspects of animal grouping: Swarms, schools, flocks, and herds," *Advances in Biophysics*, vol. 22, pp. 1 – 94, 1986.

[6] G. Flierl, D. Grnbaum, S. Levins, and D. Olson, "From individuals to aggregations: the interplay between behavior and physics," *Journal of Theoretical Biology*, vol. 196, no. 4, pp. 397 – 454, 1999.